

Su-Jin Kim · Dong-Yoon Lee · Hyun-Chul Kim · Sung-Gun Lee · Min-Yang Yang

CL surface deformation approach for a 5-axis tool path generation

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Abstract In this paper, the 5-axis tool path that has been generated on the cutter contact (CC) surface is generated on the cutter location (CL) surface, and the CL surface deformation approach that inversely deforms the 3-axis tool path generated on the deformed CL surface to a 5-axis tool path is introduced. The CL point computation and interference check based on the CL surface is faster and more robust than that based on the CC surface. The proposed CL surface deformation approach can be used if the orientation of the cutter is predefined. By the CL surface deformation approach, the 5-axis tool path generation time can be reduced to that of a 3-axis, since the complexity of a CL surface deformation is linear and because the 3-axis tool path generation and gouge removal algorithms are used at the deformed CL surface.

Keywords CL surface · Deformation · 5-axis tool path · Triangular mesh

1 Introduction

The two principle reasons for using 5-axis machining are improved surface finish and better tool accessibility. The faster machining times and superior surface finishes characteristic of 5-axis machining are achieved by using flat and rounded endmills instead of a ball endmill [1–5]. In addition, the complex multiple setups necessary in conventional 3-axis machining can be done more efficiently with a single setup [6]. However, a drawback of using 5-axis tool path generation is the complexity involved for the interference check between the cutter and the model which, until now, has made the computation time much longer than that of 3-axis machining.

In the conventional 5-axis tool path generation method, the cutter contact (CC) points are computed on a designed model. The local gouge and global interference between the tool assembly and the part model are checked for each posture of the tool on the CC points and select the feasible orientations. The means to select an interference-free orientation include the bounding volume method [7, 8], the C-space method [9–11], and visibility analysis [12–14]. A common process that should be performed in the course of these methods is collision detection between the cutter and the material. Though the regions for collision checking can be localized by bounding volume and space subdivisions, it is still a time-consuming process.

In 3-axis machining, a CL surface is widely used for generating an interference free tool path [15–17]. The offset method of a triangular mesh model is also used for tool path generation and for rapid prototyping [16, 18–20]. However, these methods are not yet well adapted to 5-axis machining, since the CL volume of either a flat or a rounded cutter does not guarantee gouge avoidance [21]. Nevertheless, in the case of a ball endmill, a CL surface can be effectively used for a 5-axis tool path generation, because the offset surface is constant, independent of tool orientation.

In this paper, the 5-axis tool path, which has been generated on the cutter contact (CC) surface, is generated on the cutter location (CL) surface for a ball endmill. The designed model is inputted by means of stereo lithography (STL) format and the CL surface, also expressed by triangular mesh, is computed by offsetting the designed mesh by the amount of the cutter radius. CL points are computed by slicing the CL surface, and the accessible orientations for the tool are selected by using collision checking between the cutter and the designed model. The collision between the cylindrical cutter and the designed model is replaced with that of the axis line and the CL surface, to reduce the complexity of the interference check.

In many practical 5-axis machining methods, the orientation of the cutter is predefined by a point or curve. Here, the infinite line of the cutter axis goes through both the CL point of the designed model and the prescribed point or curve. This machining mode is useful for machining cavities with a negative

S.-J. Kim · D.-Y. Lee · H.-C. Kim · S.-G. Lee · M.-Y. Yang (✉)
Department of Mechanical Engineering,
Korea Advanced Institute of Science and Technology,
305-701 Daejeon, South Korea
E-mail: myyang@kaist.ac.kr
Tel.: +82-42-8693264
Fax: +82-42-8693210

slope or tube geometry, such as a tire mold. Since the degrees of freedom are limited to three, by a user-defined tool orientation mode, it may be possible that effective 3-axis tool path generation and gouge avoidance algorithms can be used for 5-axis machining.

The CL surface deformation approach proposed in this paper for 5-axis tool path generation can be used if the orientation of the cutter is predefined. The CL surface is deformed to a new coordinate system where the prescribed tool orientation is arranged parallel to the z-axis like in 3-axis machining. The 3-axis tool path that is generated on a deformed CL surface using conventional algorithms is inversely deformed to a 5-axis tool path. The algorithms are implemented on a PC by means of the C++ language and tested by means of 5-axis NC machining, using the tool path generated by the CL deformation approach.

2 CL surface based 5-axis tool path generation

The CL surface based tool path generation is widely used in 3-axis machining, but it is rarely applied to 5-axis tool path generation. In this section, we discuss a CL surface based 5-axis tool path generation implemented in three steps. Firstly, the CL surface is computed by offsetting the design model by the amount of the cutter radius and allowance. Secondly, CL points of a ball endmill, with a default orientation computed by the feed direction and a normal vector of surface, are computed on the CL surface. Thirdly, the cutter interference free orientation is selected by the collision detection between the axis line of the cutter and the offset surface, which is a simpler process than a collision check between the cutter model and the design model.

2.1 Offset to the CL surface

In 5-axis machining, where the cutter orientation is flexible, the CL points of a design surface are located in the CL volume, which is bounded by two offset surfaces [21]. The two surfaces for a rounded endmill are generated by offsetting the design surface by the amount of the cutter radius and the corner radius. Therefore, most 5-axis tool path generation systems are based on the CC surface.

However, for a ball endmill, the two offset surfaces surrounding the CL volume are exactly the same surface, which is unrelated to the cutter orientation, because the cutter radius of the ball endmill is equal to the corner radius. The CL volume is reduced to the CL surface, which is generated by offsetting the design surface by the amount of the cutter radius. The triangular mesh offset for the ball endmill is implemented in accordance with a previous study, using multiple normal vectors of vertex [20].

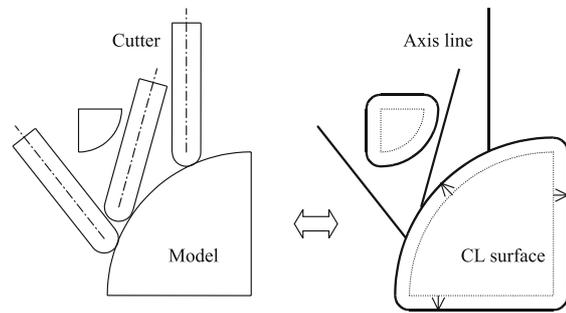
2.2 CL point and initial orientation

Since the CL points are located on the CL surface, the tool path curve is computed by slicing the CL surface by a series of planes [16-18]. The initial cutter orientations are set to have a tilting angle for a good surface quality.

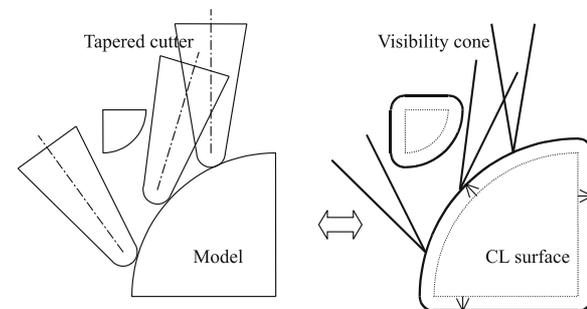
2.3 CL surface based interference check

The direct collision check between the tool assembly and the designed model has been used for accessible tool orientation selection in the CC surface based 5-axis tool path generation method [7-14]. Though the regions for collision checking can be localized by bounding volume and space subdivisions, the collision detection between the cutter and the material is still a time-consuming process.

In this section, the collision free accessible directions of tool assembly are computed using the CL surface, to reduce the time complexity of the collision check. The cutter and holder of the NC machine are composed of a straight or tapered cylinder. The global interference of the cylindrical cutter and the model is the same as that of the axis line of the cutter and the CL surface, as shown in Fig. 1a. If the cutter is located outside of the model, the line is also located outside of the CL surface. If the cutter has contact with the model, the axis line also has contact with the CL surface. If a part of the cutter interferes with the model, a part of the line also interferes with the CL surface. The interference between the tapered cutter and the model is equal to the interference between a cone with the same taper angle and the CL surface, as shown in Fig. 1b. Therefore, the interference of the ball endmill with a cylindrical or tapered shank can be changed to a visibility problem of the CL surface.



(a) Interference of cutter and model is that of axis line and CL surface



(b) Interference of tapered cutter and model is that of visibility cone and CL surface

Fig. 1a,b. CL surface based interference check **a** Interference of cutter and model is that of axis line and CL surface **b** Interference of tapered cutter and model is that of visibility cone and CL surface

The global interference of the holder and the part surface is usually checked by a machining simulation. Occasionally, collision between the holder and the part surface is checked using a CAM system; however, this is also done after CL points and postures are selected. Since the holder is also cylindrically shaped, the interference between the holder and the surface can be simplified to a contact problem between the holder axis line and the holder location (HL) surface. The HL surface is also computed by offsetting the model by the amount of the radius of the holder.

The collision check between a cutter and a designed model in the CC surface based method is simplified to that of a line and a CL surface in the CL surface based method. The complexity of interference checking between the line and the three-dimensional object is much simpler than the complexity of the interference checking among three-dimensional objects. Although there have been many attempts to reduce the complexity of the interference checking between three-dimensional objects, such as bounding volume methods [7, 8], the interference checking between a line and a three-dimensional object uses faster algorithm. Therefore, the interference check using the axis line and the CL surface is simpler and faster than the method of direct interference checking.

2.4 Post processing and simulation

The tool path is generated by joining the CL points with collision free orientations to the series of lines. Two tool path blocks are connected to the series of lines, if the end point and start point are in contact and the angle between orientations is small. If the end points are not in contact, or if the angle between orientations is large, the tool moves up to a safe height and approaches the next CL points. The tool paths are verified by material removal simulation, using three-dimensional volume elements (voxels) [22, 23].

2.5 Examples

The CL surface based 5-axis tool path generation system is developed using C++ and Open GL library. The system inputs an STL file, which is designed and tessellated on a conventional CAD system and outputs an interference-free 5-axis tool path.

The hollow sphere with three holes shown in Fig. 2a should be machined using a 5-axis machine tool, where the feasible orientation of the tool path is selected by means of interference checking. A triangular mesh with 4445 faces, generated by a commercial CAD system, is shown in Fig. 2a. The mesh is offset by the amount of the tool radius 10 mm, as shown in Fig. 2b [20]. The CL points of the tool path are directly generated by slicing the CL surface using a series of horizontal planes, and the feasible orientations of the cutter are selected by means of the interference check between the cutter axis line and the CL surface. The tool path line and the cutter orientation vectors of the resulting tool path are shown in Fig. 2c. The computation time with PC (Pentium 4 CPU 2.4 GHz and 512 MB memory) takes about 146 s. The simulation result, using a voxel model in Fig. 2d revealed no global interference.

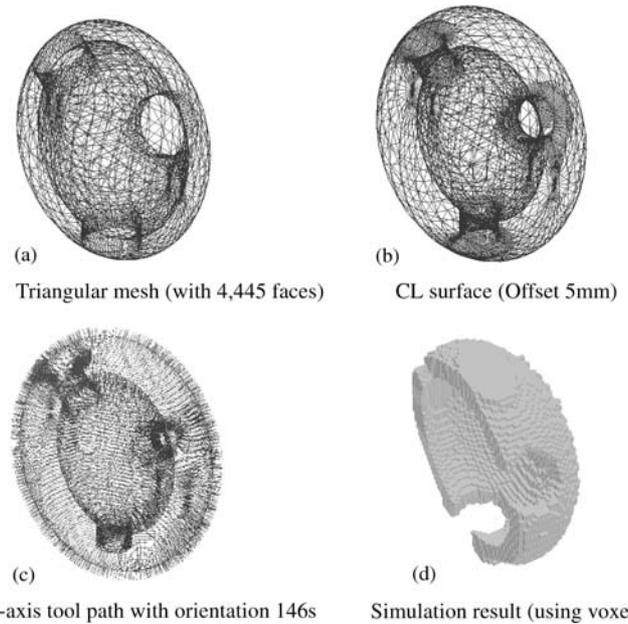


Fig. 2a–d. CL surface based 5-axis tool path of hollow sphere **a** Triangular mesh (with 4,445 faces) **b** CL surface (Offset 5mm) **c** 5-axis tool path with orientation (146 s) **d** Simulation result (using voxel)

The 5-axis tool path for the model of a handle is generated and an interference free orientation is automatically computed from the CL surface, as shown in Fig. 3. A triangular mesh with 223 779 faces is generated using a commercial CAD system with a 0.01 mm tolerance. Figure 3a shows the designed model and the ball endmill with a 6 mm diameter. The interference between the model and the cutter is the same as the interference between the CL surface and the axis line, as shown in Fig. 3b. The 5-axis tool path and interference free orientations are shown in Fig. 3c. The total computation time is 25 s, which is shorter than any con-

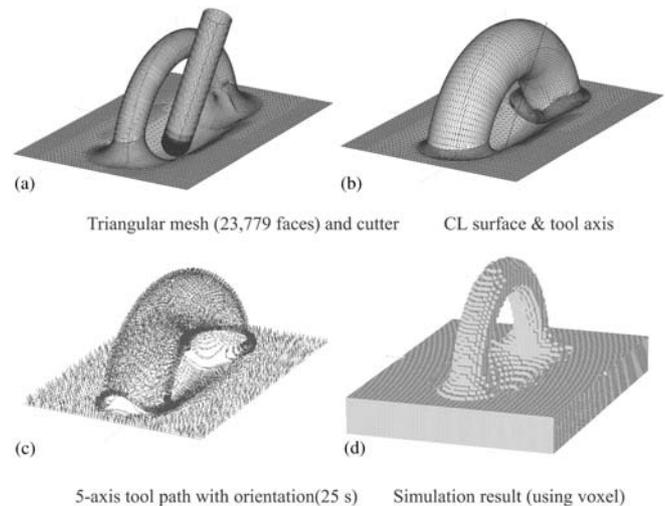


Fig. 3a–d. CL surface based 5-axis tool path of handle **a** Triangular mesh (23,779 faces) and cutter **b** CL surface & tool axis **c** 5-axis tool path with orientations (25 s) **d** Simulation result (using voxel)

ventional method, because of the CL surface based interference check method. The simulation results shown in Fig. 3d prove that the tool path generated using a CL surface based method is interference free.

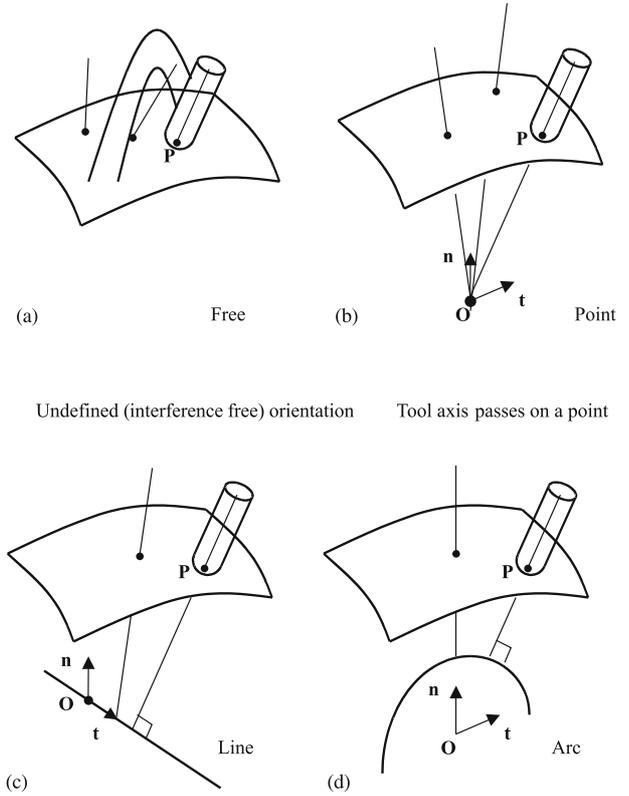
2.6 Results and discussion

A CL surface based 5-axis tool path generation method for a ball endmill was implemented. The local and global interference checking of the CL surface based method was much simpler than that of the CC surface based method. The CL surface based 5-axis tool path generation method is faster than the CC surface based method; however, it is still much slower than a 3-axis tool path generation. The largest disadvantage of the CL surface based 5-axis tool path generation is that it can only be applied for a ball endmill.

3 CL surface deformation approach

Though the CL surface based 5-axis tool path generation is simpler than the CC surface based method, the computation time is still much longer than a conventional 3-axis tool path generation and is applicable only for the ball endmill. Though the collision check between a cutter and a model was simplified to that of a cutter axis and a CL surface, the most time-consuming step is still the collision checking for a feasible orientation selection. The CL surface of a flat or rounded endmill cannot be defined before the cutter orientation is selected. The long computation time and the limitation of cutter geometry in a CL surface based 5-axis tool path generation are caused from an unselected cutter orientation. Many 5-axis tool path generation systems are defining the cutter orientation before a tool path generation. The conventional orientation definition methods involve using a reference point, a line, and an arc, through which the cutter axis goes, as shown in Fig. 4. Although manually defining an orientation before the tool path generation reduces flexibility, the predefined orientation is effective for a 5-axis tool path generation for cylindrical, spherical, and tube geometry.

The CL surface deformation approach is based on the concept that a 5-axis tool path can be generated using a conventional 3-axis tool path generation method, if a predefined cutter orientation reduces the degrees of freedom to three. In this approach, the CL surface of 5-axis machining is deformed to the CL surface of 3-axis machining and the 3-axis tool path generated on the deformed CL surface is inversely deformed to the 5-axis tool path, as shown in Fig. 5. The CL surface deformation approach consists of four steps. Firstly, the CL surface is computed by offsetting the designed model by the amount of the tool radius and allowance. Secondly, the CL surface is deformed to the new coordinates where the tool axis is parallel to the z-axis, as in 3-axis machining. Thirdly, the tool path is computed by slicing the deformed CL surface using a series of planes. A conventional 3-axis tool path generation and gouge avoidance method can be applied to a tool path generation on a deformed CL surface. Finally, the 3-axis tool path in a deformed coordinate system is



Tool axis meets perpendicular to a line Tool axis meets perpendicular to an arc
Fig. 4a–d. Orientation definition methods **a** Undefined (interference free) orientation **b** Tool axis passes on a point **c** Tool axis meets perpendicular to a line **d** Tool axis meets perpendicular to an arc

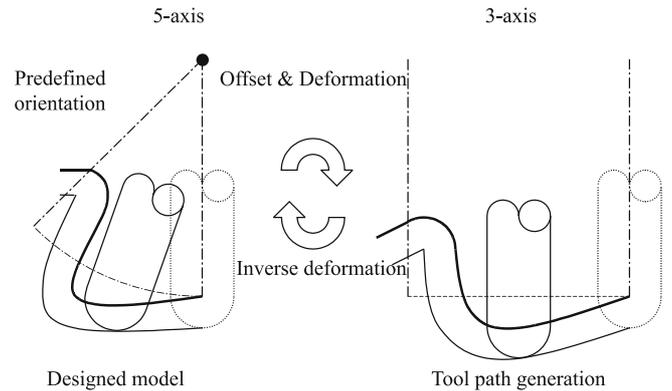


Fig. 5. Offset deformation for the 5-axis tool path generation with predefined orientation

inversely deformed to the 5-axis tool path in the original coordinate system.

3.1 Offset to the CL surface

The first step of the CL surface deformation approach for tool path generation is a CL surface computation, similar to other

common CL surface based tool path generation methods. The CL surface of the ball endmill, which is the offset surface of the original mesh, is computed by the mesh offset method using multiple normal vectors of vertices [20]. The CL surface of the flat or rounded endmill is not unique to 5-axis machining, since the CL point of a CC point changes according to the cutter orientation. However, if the cutter orientation is predefined before offsetting, the CL surface of a flat or rounded endmill is unique, as shown in Fig. 6b. Therefore, a CL surface based on a 5-tool path generation is possible for the flat or rounded endmill, if the orientation is predefined.

3.2 Deformation

If the tool orientation is predefined, the CL surface is deformed to use a conventional 3-axis tool path generation and gouge avoidance method. The deformation function is used to change the coordinates so that the orientation of the tool is parallel to the z-axis in the deformed coordinate system. If the orientation is defined to go through a point, the deformation function is the same as the coordinate transformation of spherical coordinates to orthogonal coordinates. If the orientation is defined to meet perpendicular to a line, the deformation is the same as the coordinate transformation of a cylindrical coordinates to orthogonal coordinates. The CL points \mathbf{P} in the original coordinates are translated to the CL points \mathbf{P}' in the deformed space by the deformation functions \mathbf{T} , as shown in Eq. 1.

$$\mathbf{P}' = \mathbf{T}\mathbf{P} \quad (1)$$

There are various deformation functions, corresponding to various tool orientation definition methods. If the cutter orientation is defined by the point on which the tool axis always passes, as shown in Fig. 4b, the CL point in the original space is transformed to a new CL point in the deformed space, according to Eq. 2. The reference radius R is the average distance from a point to the CL surface and \mathbf{p} is the vector from the point \mathbf{O} to the CL point \mathbf{P} .

$$\mathbf{P}' = \begin{pmatrix} R\Phi \\ R\Theta \\ |\mathbf{p}| \end{pmatrix} \quad (2)$$

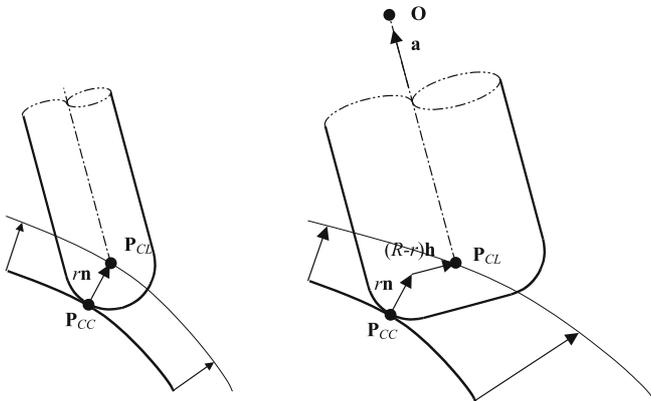


Fig. 6. Offset to CL surface with predefined orientation

where,

$$\Phi = \tan^{-1} \left(\frac{\mathbf{p} \cdot (\mathbf{t} \times \mathbf{n})}{\sqrt{(\mathbf{p} \cdot \mathbf{t})^2 + (\mathbf{p} \cdot \mathbf{n})^2}} \right), \quad \Theta = \tan^{-1} \left(\frac{\mathbf{p} \cdot \mathbf{t}}{\mathbf{p} \cdot \mathbf{n}} \right)$$

If the cutter orientation is defined by the line which the tool axis always meet perpendicular to, as shown in Fig. 4c, the CL point is transformed, as shown in Eq. 3.

$$\mathbf{P}' = \begin{pmatrix} \mathbf{p} \cdot \mathbf{t} \\ R\Theta \\ |\mathbf{p} - (\mathbf{p} \cdot \mathbf{t})| \end{pmatrix} \quad (3)$$

where,

$$\Theta = \tan^{-1} \left(\frac{\mathbf{p} \cdot (\mathbf{t} \times \mathbf{n})}{\mathbf{p} \cdot \mathbf{n}} \right)$$

If the cutter orientation is defined by a circle (or arc), which the tool axis always meets perpendicular to, as shown in Fig. 4d, the CL point is transformed, as shown in Eq. 4. The reference radius R_o is the average distance from the circle with radius R_i to the CL surface.

$$\mathbf{P}' = \begin{pmatrix} (R_i + R_o)\Theta \\ R_o\Phi \\ |\mathbf{p} - \mathbf{c}| \end{pmatrix} \quad (4)$$

where,

$$\Theta = \tan^{-1} \left(\frac{\mathbf{p} \cdot \mathbf{t}}{\mathbf{p} \cdot \mathbf{n}} \right),$$

$$\mathbf{c} = R_i (\sin \Theta \mathbf{t} + \cos \Theta \mathbf{n}),$$

$$\Phi = \tan^{-1} \left(\frac{(\mathbf{p} - \mathbf{c}) \cdot (\mathbf{t} \times \mathbf{n})}{\sqrt{((\mathbf{p} - \mathbf{c}) \cdot \mathbf{t})^2 + ((\mathbf{p} - \mathbf{c}) \cdot \mathbf{n})^2}} \right)$$

3.3 CL point generation

Since the orientation of the cutter is the same as that of 3-axis machining in a deformed coordinates, a conventional CL surface based 3-axis tool path generation method can be used to generate the tool path in deformed coordinates. When the CL surface is a triangular mesh, the tool path is generated by slicing the surface using a series of planes or by projecting curves to the surface [16, 17]. The interference is removed after the slicing, by comparing the heights of the overlapped tool paths; this is a much faster method than the conventional interference check of a 5-axis tool path.

3.4 Inverse deformation

Since the 3-axis tool path is the points on the deformed CL surface, the 5-axis tool path is calculated by an inverse deformation of the 3-axis tool path.

$$\mathbf{P} = \mathbf{T}^{-1}\mathbf{P}' \quad (5)$$

If the cutter orientation is defined by the point which the tool axis always passes on, as shown in Fig. 4b, the CL point in the deformed space is inversely transformed to the CL point in the original space, as shown in Eq. 6.

$$\mathbf{P} = \mathbf{O} + P'_z ((\cos \Phi \cos \Theta)\mathbf{n} + (\cos \Phi \sin \Theta)\mathbf{t} + \sin \Phi(\mathbf{t} \times \mathbf{n})) \quad (6)$$

where,

$$\Phi = \frac{P'_y}{R}, \quad \Theta = \frac{P'_x}{R}$$

If the cutter orientation is defined by the line which the tool axis always meets perpendicular to, as shown in Fig. 4c, the CL point is inversely transformed, as shown in Eq. 7.

$$\mathbf{P} = \mathbf{O} + P'_x \mathbf{t} + P'_z (\cos \Theta \mathbf{n} + \sin \Theta (\mathbf{t} \times \mathbf{n})) \quad (7)$$

where,

$$\Theta = \frac{P'_y}{R}$$

If the cutter orientation is defined by the circle (or arc) to which the tool axis always meets perpendicular as shown in Fig. 4d, the CL point is inversely transformed, as shown in Eq. 8.

$$\mathbf{P} = \mathbf{O} + (R_i P'_x \cos \Phi) (\cos \Theta \mathbf{t} + \sin \Theta \mathbf{n}) + P'_z \sin \Phi (\mathbf{t} \times \mathbf{n}) \quad (8)$$

where,

$$\Theta = \frac{P'_x}{R_+ R_o}, \quad \Phi = \frac{P'_y}{R_o}$$

3.5 Deformation error

The CL surface deformation and the tool path inverse deformation increase machining error because the triangle faces and the tool path lines are simplified faces and lines, regardless of deformation. In fact, a triangle face in the original space is a curved surface in the deformed space and the linear tool path in the deformed space is a curved tool path in the inversely deformed space. To decrease the deformation error, the maximum length d of the edge and the tool path block need to be limited by the allowance δ and the reference radius R of deformation, as shown in Eq. 9.

$$d \leq 2\sqrt{2R\delta} \quad (9)$$

The maximum edge length of the triangular mesh is limited at tessellation in a CAD system and the maximum block length of a tool path is limited at the tool path generation step in deformed space.

3.6 Examples

The CL surface deformation approach for 5-axis machining is implemented by the C++ language. A triangular mesh is input to the system and is offset to the CL surface. If the cutter orientation

is defined before tool path generation, a CL surface deformation approach is used to simplify the 5-axis tool path generation.

3.6.1 Cooling pin

A cooling pin is small aluminum part used to remove the heat from the chips of the computer. If wings of the pin are in contact with the cylindrical pipe, the cooling pin needs to be a machine in the 5-axis machine tool. Figure 7a shows a predefined tool orientation that the tool axis always passes perpendicular to the center line of the cylinder. The y-axis of the coordinates is the center line of the cylinder, the x-axis is the horizontal direction, and the z-axis is the vertical direction. The rotation angle θ is the angle

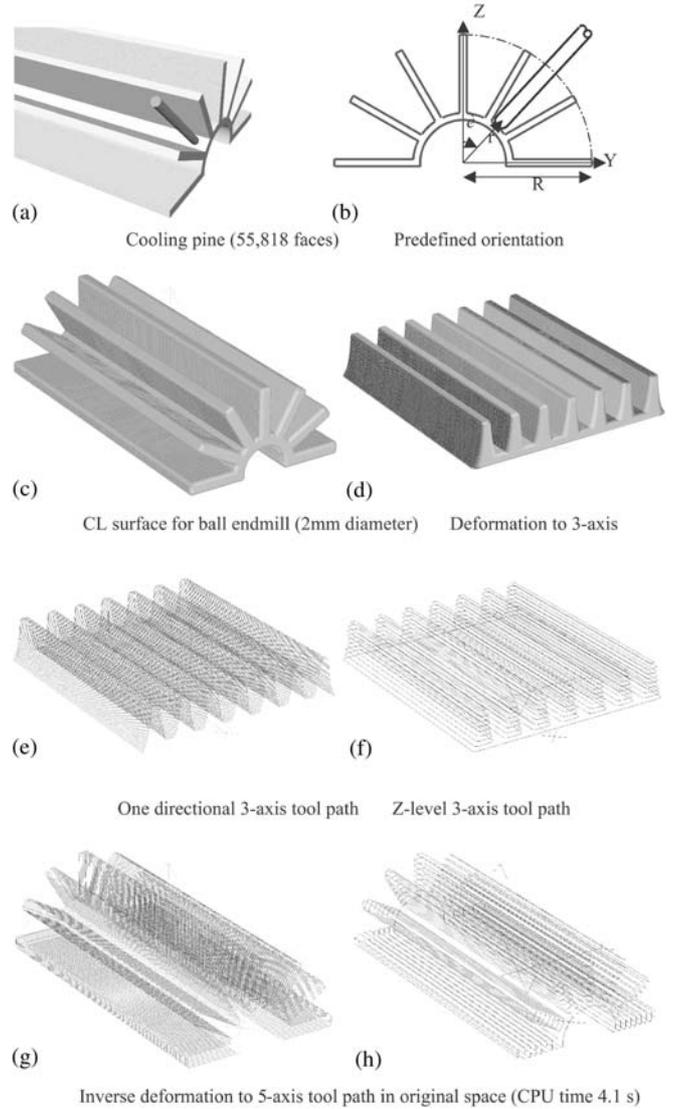


Fig. 7a–h. 5-axis tool path of cooling pin (CL surface deformation method) **a** Cooling pin (55,818 faces) **b** Predefined orientation **c** CL surface for ball endmill (2 mm diameter) **d** Deformation to 3-axis **e** One directional 3-axis tool path **f** Z-level 3-axis tool path **g, h** Inverse deformation to 5-axis tool path in original space (4.1 s)

from the z-axis to the axis line of the cutter and r is the distance from the y-axis line to the reference point of the ball endmill. Since the tool orientation is dependent on the position of the tool, the degrees of freedom are reduced to three, and the CL surface can be deformed to use 3-axis tool path generation and gouge avoidance methods.

The x position in the deformed space is the same as the original x-axis, the y position is the length of the arc with radius R , and the z position is the distance from the x-axis to the reference point of the endmill. Equation 10 shows the coordinate deformation function used to deform the CL surface of the cooling pin model.

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} X \\ R\Theta \\ r \end{pmatrix} \quad (10)$$

where,

$$r = \sqrt{X^2 + Y^2}, \quad \Theta = \tan^{-1} \left(\frac{Y}{X} \right)$$

The inverse deformation function is shown in Eq. 11.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X' \\ r \sin \Theta \\ r \cos \Theta \end{pmatrix} \quad (11)$$

where,

$$r = Z', \quad \Theta = \frac{Y'}{R}$$

A STL file with 55 818 faces, 0.01 mm tolerance, and a 2.0 mm maximum edge length is outputted by a conventional CAD system, as shown in Fig. 7b. The CL surface for a ball endmill with a 2 mm diameter is computed by offsetting the triangular mesh, as shown in Fig. 7c. The CL surface, which is a triangular mesh, is deformed according to Eq. 10. The deformed mesh in Fig. 7d is the CL surface for a 3-axis tool path generation, where the tool orientation is the same as the z-axis. The one-directional tool path and the constant z-level tool path are generated by slicing the deformed mesh and removing the invalid portion of the CL point, as shown in Fig. 7e,f. The 3-axis tool path in the deformed space is inversely deformed to the original space, as shown in Fig. 7g,h. The resultant 5-axis tool path is interference free and the total computation time is 4.1 s, which is slightly longer than the 3-axis tool path generation.

3.6.2 Segmented mold of tire

The geometry of a tire is like a torus, and a tire mold is divided to a side mold and a segmented mold. Because of the tire geometry, the mold must be segmented, and each segmented mold must be machined by a 5-axis NC machine [24, 25]. The aluminum stock with a cylindrical geometry is machined by the turning center, and threads and grooves on segmented mold are machined by 5-axis machining. The orientation of the tool axis is selected so that

the axis line is normally always in contact with the center circle of a torus, as shown in Fig. 8a.

In the deformed space, the x-axis is defined by the outer radius ($R_i + R_o$) and the orientation angle θ . The y-axis is defined by the orientation angle ϕ and the reference radius R_o , and the z-axis is defined by the distance from the circle to the CL point, as

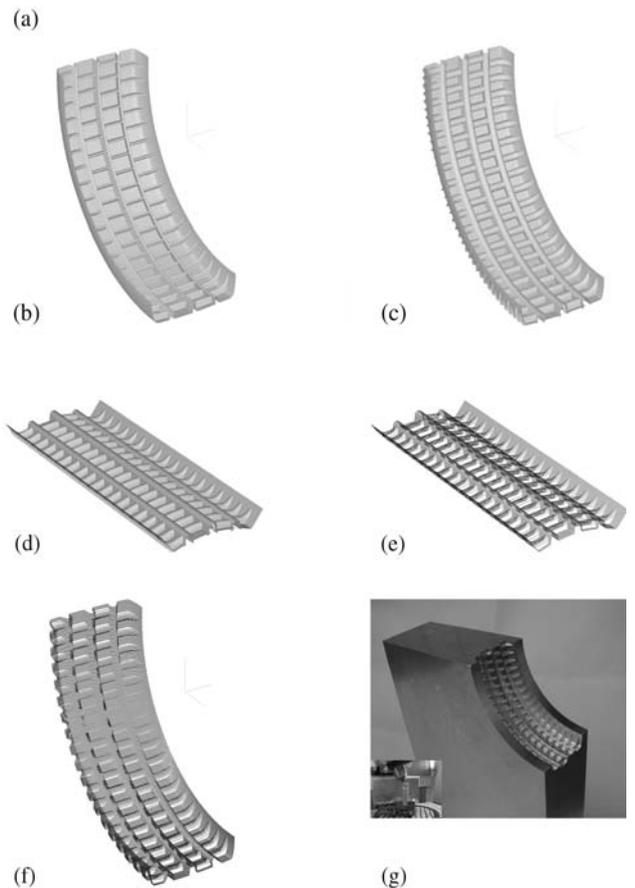
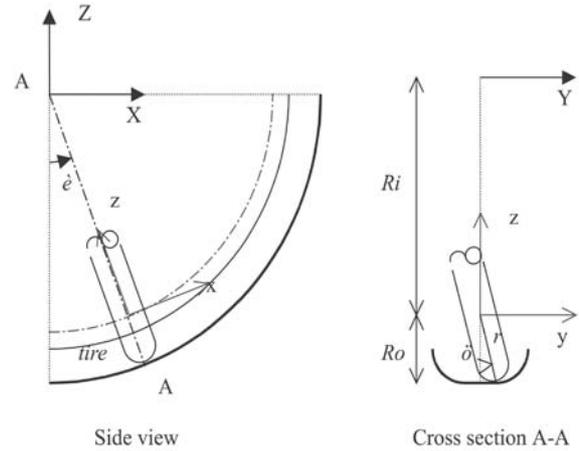


Fig. 8a-g. 5-axis tool path of tire mold (CL surface deformation method) **a** Predefined orientation **b** STL model with 236,748 faces **c** Offset for ball endmill(2 mm diameter) **d** Deformation to 3-axis **e** 3-axis tool path generation **f** Inverse deformation to 5-axis tool path (16.6 s) **g** Machined part

shown in Eq. 12.

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} (R_i + R_o)\theta \\ R_o\Phi \\ R_o - r \end{pmatrix} \quad (12)$$

where,

$$\begin{aligned} \Theta &= \tan^{-1} \left(\frac{X}{-Z} \right), \\ \Phi &= \tan^{-1} \left(\frac{y}{Z \cos \Theta - X \sin \Theta} \right), \\ r &= \sqrt{(Z \cos \Theta - X \sin \Theta + R_i)^2 + Y^2} \end{aligned}$$

The inverse deformation function that transforms the tool path in the deformed 3-axis space to the original 5-axis space is shown in Eq. 13.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (R_i + r \cos \Phi) \sin \theta \\ r \sin \Phi \\ -(R_i + r \cos \phi) \cos \Theta \end{pmatrix} \quad (13)$$

where,

$$\Theta = \frac{X'}{R_i + R_o}, \quad \Phi = \frac{Y'}{R_o}, \quad r = R_o - Z'$$

A STL file with 236 748 faces, 0.01 mm tolerance, and a 2.0 mm maximum edge length is outputted by a conventional CAD system, as shown in Fig. 8b. The CL surface for the ball endmill with a 2 mm diameter is computed by offsetting the triangular mesh, as shown in Fig. 8c. The CL surface in 5-axis is deformed to the CL surface in 3-axis where tool orientation is the same as the z-axis as shown in Fig. 8d. A one-directional tool path is generated by slicing the deformed CL surface and removing the invalid portion of the CL point, as shown in Fig. 8e. The 3-axis tool path is inversely deformed to the original space, as shown in Fig. 8f. The resultant 5-axis tool path is interference free and the total computation time using a PC is 16.6 s, which is similar to the time for a 3-axis tool path generation.

The tool path is post processed for the 5-axis machine of table-rotating and head-tilting type [26]. The 3-axis tool path is used for roughing and the 5-axis tool path generated by means of the CL surface deformation method is used for both semi-finishing and finish cutting. Figure 8g shows the machine cutting the aluminum alloy stock and the machined result. The cutter used for finishing is a ball endmill with a 2 mm diameter and the side step of the tool path is 0.2 mm. The finish machining time took about 80 min, and there is no over cut at the final surface.

3.7 Results and discussion

If the orientation is predefined by a user, the proposed CL surface deformation method can be used for an effective 5-axis tool path generation. The advantage of the proposed method is that well-developed 3-axis tool path generation algorithms are applied to 5-axis tool path generation simply by using the defor-

Table 1. Comparison of computation times

Model	Faces	CL surface based 5-axis	CL surface deformation 5-axis	3-axis tool path
Sphere	4,445	146 s		0.6 s
Handle	23,779	25 s		1.4 s
Pin	55,818		4.1 s	3.5 s
Tire	236,748		16.6 s	12.8 s

mation of the CL surface. In addition, the tool path generation time will be almost the same as that of the 3-axis tool path generation, because the deformation of triangular mesh and the inverse deformation of the tool path are very fast. The complexity of deformation is $O(n)$, where n is the number of vertices, and the complexity of inverse deformation is $O(m)$, where m is the number of the tool path block. However, the flexibility of the tool path is lower than that of other methods, because the orientation of the axis is defined before the tool path generation.

Table 1 compares the computation time of the CL surface based 5-axis tool path generation and the CL surface deformation approach to that of a 3-axis tool path generation.

The tool paths for a hollow sphere and a handle model are generated by the CL surface based 5-axis tool path generation method. Although the interference check method between the cutter and the model is much simpler than that of the CC surface based method, the 5-axis tool path generation time is still much longer than that of a 3-axis tool path generation.

The CL surface deformation method is used to generate a tool path for a cooling pin and a segmented tire mold. Since the complexity of the deformation of triangular mesh and the inverted deformation of the tool path is linear, the computation time of a 5-axis tool path generation is similar to that of a 3-axis tool path generation.

It shows that the CL surface deformation method is as fast as a conventional 3-axis tool path generation. However, the CL surface based 5-axis tool path generation method also needs to be used, since the CL surface deformation method can be used when the orientation is predefined.

4 Conclusions

In this study, a CL surface based 5-axis tool path generation was implemented and a new CL surface deformation approach is introduced. The CL surface is computed by offsetting a triangular mesh by the sum of the tool radius and cutting allowance. The CL points of the tool path are generated by slicing the CL surface and the interference free orientations are selected by conducting a collision check between the axis line and the CL surface. Though CL surface based 5-axis tool path generation is much faster than the CC surface based method, the computation time is still longer than that of the 3-axis tool path generation and is applicable only for a ball endmill.

If the orientation of the tool is predefined by a point or a curve, a 5-axis tool path can be generated using the pro-

posed CL surface deformation method and conventional 3-axis tool path generation algorithms. In the CL surface deformation method, the predefined orientations of the cutter axis are transformed so that they are parallel to the z-axis in the deformed space. After deformation, fast and effective 3-axis tool path generation and gouge removal methods can be used, and the resulting tool path is inversely deformed to the original space. Since the complexity of offsetting and deformation and inverse deformation algorithms used in this paper is $O(n)$, where n is the number of faces on the mesh, the computation time is nearly identical to or slightly longer than that of 3-axis tool path generation. The proposed 5-axis tool path generation procedure can be practically applied to a CAM system that uses triangular mesh and has a CL surface based 3-axis tool path generation algorithm.

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