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ORIGINAL ARTICLE

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A CL surface deformation approach for constant scallop height tool path generation from triangular mesh

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Abstract In this paper, we present a cutter location (CL) surface deformation approach for constant scallop height tool path generation from triangular mesh. The triangular mesh model of the stereo lithography (STL) format is offset to the CL surface and then deformed in accordance with the deformation vectors, which are computed by the slope and the curvature of the CL surface. In addition, the tool path, which is computed by slicing the deformed CL surface, is inversely deformed by those same deformation vectors to a tool path with a constant scallop height. The proposed method is implemented, and a tool path is generated and tested by simulation and by numerical control (NC) machining. The scallop height was found to be constant over the entire machined surface, demonstrating much better quality than that of mesh slicing, under the same constraints for machining time.

Keywords CL surface deformation · Constant scallop height · Tool path · Triangular mesh

1 Introduction

There are three kinds of tool path generation methods according to the cutter location (CL) curve computation algorithms from the surface: iso-parametric, iso-plane and constant scallop height.

In the iso-parametric tool path generation method, the cutter contact (CC) points are computed by uniformly increasing the parametric value of the part surface, and the CL points are computed by offsetting the CC points. This method is useful for the finish machining of a single parametric surface, but there are gouge problems for models with multiple surfaces.

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The iso-plane tool path generation is most commonly used for three-axis tool path generation. The tool path is computed by slicing the designed surface or the offset surface by using a series of planes. If the slicing plane is parallel to the xy-plane, it is called a "constant z-level" tool path; and if the slicing plane is a vertical plane it is called a "one-way" or "zigzag" tool path. With this method, the scallop height increases sharply when the slope of part surface is large. To limit the scallop height, the interval of slicing planes is changed, or new slicing planes are inserted on the region with a large scallop [1–3].

In the constant scallop height tool path generation approach, the path interval is determined such that scallop height is kept constant for uniform machining quality and the shortest tool path length [4–12]. The initial tool path is defined by either the center line or the boundary curve of model, and path intervals to the adjacent tool paths are selected so that the scallop height is kept constant; the next tool path is computed by offsetting the current paths by the amount of the computed intervals. In most cases, the constant scallop height tool path has been generated from a parametric surface format model. The drawback of the parametric surface model is the complexity of the cutter gouge checking, which is one of the most critical problems in generating a tool path from a compound surface.

Recently, a triangular mesh model, which is superior to the parametric surface in terms of gouge checking, has been used in commercial computer aided machining (CAM) software and research. The triangular mesh model is more popular as the STL format, which is provided by most CAD systems for rapid prototyping (RP) and NC machining. An iso-plane tool path generation method from a triangular mesh model has been considered. Here, slicing consists of three steps [13, 14]: first, the triangular mesh is offset to the CL surface, which is also a triangular mesh with an invalid portion [13, 15–17]; second, a set of line segments is obtained by slicing the triangular mesh with two-dimensional geometric elements; and third, the invalid portions of the tool path generated from the invalid portions of the CL surface are removed for gouge avoidance. It is a very effective method for gouge-free tool path generation, but the scallop height sharply increases at the sloped part. In the case of triangular mesh, there has been little research considering the tool path generation with a constant scallop height. The curvature and slope of the triangular mesh must be computed by different methods to establish a parametric surface and to offset base paths on the CL surface, which is expressed by a triangular mesh with invalid portion. Needless to say, it is a complex process.

The objective of this paper is to propose a tool path generation method with a constant scallop height for the machining of triangular mesh models that are widely used in RP and NC machining. The method follows the mesh slicing method, and introduces the CL surface deformation method for constant scallop height machining. The STL file is inputted to the system and is offset by the amount of cutter radius for CL surface generation, and the deformation vectors are computed from the slope and the curvature of the CL surface. The iso-plane tool path, which is generated by slicing the deformed CL surface with a series of planes at a constant interval, is inversely deformed to the tool path with constant scallop height by deformation vectors. For example, the iso-plane tool path generated using the mesh slicing method and the constant scallop height tool path generated using the CL surface deformation method are compared by means of NC machining.

2 CL surface deformation model

2.1 Path interval

The scallop height is defined as the maximum thickness of the uncut volume. The tool path interval (or "side-step") is the distance between the two adjacent paths. When the radius of curvature of the part surface along the normal direction of a tool path is approximately equal to ρ, the path interval l is related to the cutter radius r and the scallop height h as follows [5, 7]:

$$l = \frac{|\rho| \sqrt{4\,(r+\rho)^2\,(h+\rho)^2 - \left[\rho^2 + 2r\rho + (h+\rho)^2\right]^2}}{(r+\rho)\,(h+\rho)} \tag{1}$$

where the radius of curvature ρ is positive for a convex surface and negative for a concave surface. If the radius of curvature of surface is large relative to the cutter radius, then the surface between the two adjacent paths can be assumed to be planar. The path interval is simplified to Eq. 2 for the plane surface.

$$l = 2\sqrt{2rh - h^2}, \quad \rho \gg r \tag{2}$$

If the scallop height is much less than the cutter radius, Eq. 1 is simplified to Eq. 3, which is used to calculate a path interval. The curvature of the part surface has a positive value if it is convex and a negative value if it is concave [12].

$$l = \sqrt{\frac{8hr\rho}{\rho + r}}, \quad r \gg h \tag{3}$$

Equations 1, 2, and 3 are used when a tool path is generated on a CC surface. If the path interval L on a CL point and the curvature R of a CL surface along the normal direction to a tool path are used, then Eq. 3 is replaced with Eq. 4.

$$L = \sqrt{\frac{8hrR}{R - r}}, \quad r \gg h \tag{4}$$

The interval of the slicing plane Δ is affected by the angle θ between the horizontal plane and the part surface along the normal direction of the tool path, as shown in Eq. 5 [3]:

$$\Delta = L\cos\theta = \cos\theta\sqrt{\frac{8hrR}{R - r}} \tag{5}$$

Since the scallop height and the cutter radius are constant values, the interval of the slicing plane is a function of the slope angle and the curvature of the surface along the direction of a path interval. The interval of the slicing plane must be decreasing at the slope surface and increasing at the horizontal plane. The scallop height of the iso-plane tool path is small at the horizontal surface and large at the slope surface when the interval of the slicing plane is constant.

2.2 CL surface deformation

A tool path with a constant path interval is effective both for machining time and surface quality. The common method of constant scallop height tool path generation involves the computation of the next tool path by offsetting the base tool path on a parametric surface by the amount of the path interval on each point. However, the algorithm used to compute the path interval and the offset base tool path on parametric surface cannot be applied to a triangular mesh. Here, the mesh model is bounded by triangular faces instead of a smooth surface, and the offset mesh (the CL surface) includes many invalid portions.

The CL surface deformation method is new approach that deforms the CL surface along the normal direction of the slicing plane and inversely deforms the sliced tool paths for constant scallop height machining. The path interval of the iso-plane tool path generated by the mesh offsetting and slicing method is large

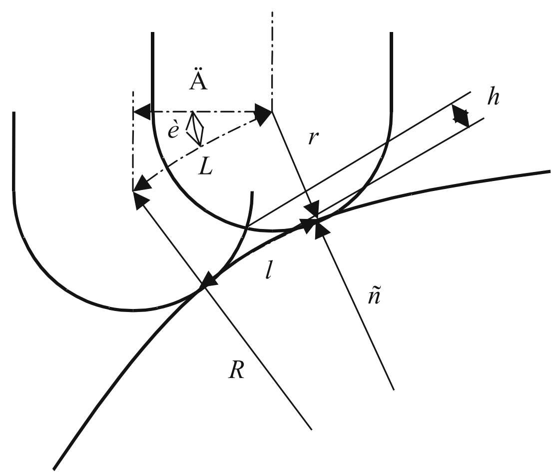


Fig. 1. The effect of slope and curvature of the design surface on scallop height

at the sloped wall and small at the plane surface, as shown in Fig. 2. In the CL surface deformation method, the sloped or concave part on a CL surface is extended in the direction of the path interval to compensate for this phenomenon. The deformation ratio of each region is computed from the slope angle and the curvature of the CL surface in the normal direction of the tool path. The tool path generated from a deformed CL surface is inversely deformed to the original space. By means of the deformation and the inverse deformation, the path interval becomes uniform, independent of the slope and the curvature of the CL surface.

The x-axis of a coordinate is set perpendicular to the slicing plane, and the y-axis is set parallel to the plane. The horizontal grid is set on the xy-plane and deformation vectors for each element are computed from the slope angle and the curvature. The z values of the mesh on a horizontal grid with interval d_x are computed, and the slope angle θ and curvature R of the mesh along the x-axis are computed from three points, as shown in Eq. 6 and Eq. 7:

$$\theta_{i,j} = \tan^{-1}\left(\frac{z_{i+1,j} - z_{i-1,j}}{2d_x}\right) \tag{6}$$

$$R_{i,j} = \frac{2z_{i,j} - \left(z_{i-1,j} + z_{i+1,j}\right)}{d_x^2} \tag{7}$$

The region with a large slope angle is extended in the normal direction of the tool path in inverse an proportion to $\cos(\theta)$, and the concave region is extended according to the ratio of the cutter radius and the radius of curvature of the CL surface. The deformation ratio ε is computed from the slope angle and the curvature of the CL surface, as shown in Eq. 8. Figure 3 shows the relationship between the deformation ratio, the slope angle, and the curvature of the CL surface over that of the cutter.

$$\varepsilon_{i,j} = \frac{1}{\cos\theta_{i,j}}\sqrt{1 - \frac{r}{R_{i,j}}} \tag{8}$$

The deformation ratio ε is calculated by combining the nearest three values along the y-axis for the smooth movement of the tool path, as shown in Eq. 9:

$$\varepsilon_{i,j} = \frac{\varepsilon_{i,j+1} + 2\varepsilon_{i,j} + \varepsilon_{i,j-1}}{4} \tag{9}$$

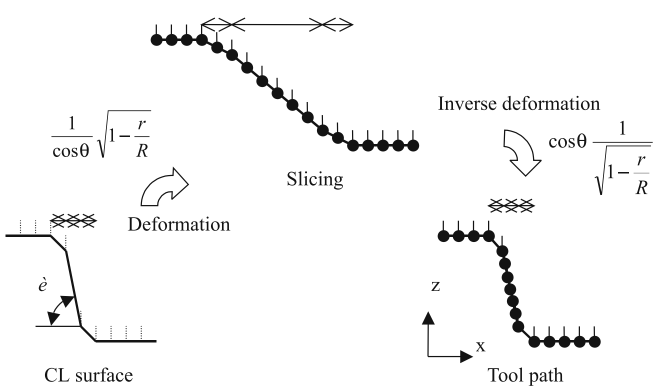


Fig. 2. CL surface deformation method

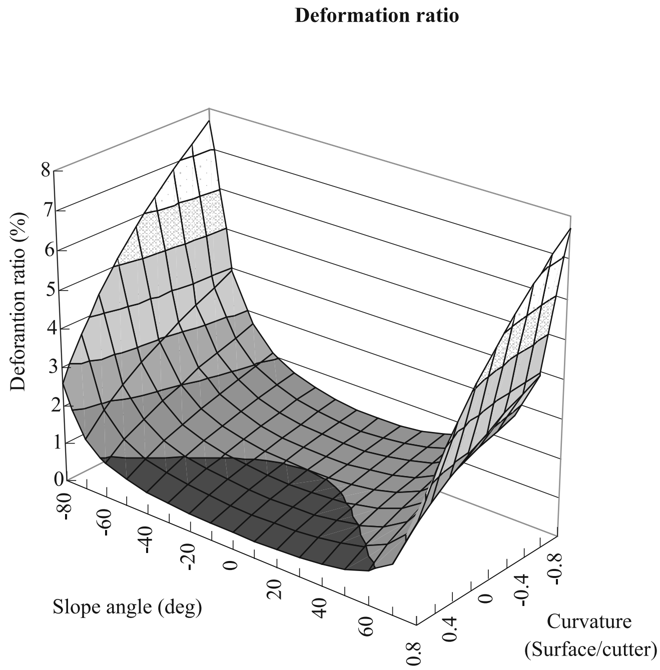


Fig. 3. Deformation ratio of the CL surface for a constant scallop height

The base line of the deformation vector, where the deformation vector is set to zero, is located in the middle of the model, such that the middle line of the model is not deformed but the left and right sides are deformed. The deformation vectors d are the weighted value of deformation.

$$d_{i,j} = \begin{cases} -\sum_{k=i}^{m-1} d_x\left(\varepsilon_{k,j} - 1\right) & i < m \\ 0 & i = m \\ \sum_{k=m+1}^{i} d_x\left(\varepsilon_{k,j} - 1\right) & i > m \end{cases} \tag{10}$$

The deformation ratio is computed on the assumption that the direction of the tool path is parallel to the y-axis. This assumption is incorrect because the resulting constant scallop height tool path is longer parallel to the y-axis. Therefore, the slope angle and the curvature along the y-axis affect the deformation ratio in the region where the CL surface undergoes shear deformation. The shear angle γ is computed from the deformation vectors computed on the above assumption, as shown in Eq. 11:

$$\gamma_{i,j} = \tan^{-1}\left(\frac{d_{i,j+1} - d_{i,j-1}}{2d_y}\right) \tag{11}$$

The compensated deformation ratio is computed from the deformation ratio along the x-axis and y-axis, ε_x and ε_y, and the shear angle, as shown in Eq. 12:

$$\varepsilon_{i,j} = \varepsilon_{xi,j}\cos\gamma_{i,j} + \varepsilon_{yi,j}\sin\gamma_{i,j} \tag{12}$$

The deformation vectors are recalculated using compensated deformation ratios and Eq. 10. The CL surface is deformed by moving all vertices of the mesh, using the deformation vectors. A vertex v of mesh is located in a grid i, and j is moved using the nearest four deformation vectors around the vertex, as shown in Eq. 13:

$$d_{\text{sum}} = d_{i,j}\,(x_{i+1} - v_x)\left(y_{j+1} - v_y\right) + d_{i+1,j}\,(v_x - x_i)\left(y_{j+1} - v_y\right) + d_{i,j+1}\,(x_{i+1} - v_x)\left(v_y - y_j\right) + d_{i+1,j+1}\,(v_x - x_i)\left(v_y - y_j\right) \quad (13)$$

$$v'_x = v_x + \frac{d_{\text{sum}}}{(x_{i+1} - x_i)\left(y_{j+1} - y_j\right)}$$

The tool path is generated from the deformed mesh by slicing planes parallel to the yz-planes. The gouge is removed by selecting the height path from the overlapped tool paths. The tool path on the deformed space is an iso-plane tool path moving along the plane parallel to the yz-plane.

The iso-plane tool path on the deformed space is inversely deformed using the deformation vectors that are used for the deformation of CL surface. A CL point p'_x of the tool path on the deformed space is moved to a CL point p_x on the original space by using the nearest four deformation vectors around the point, as shown in Eq. 14:

$$\begin{aligned} d_{\text{sum}} = {} & d_{i,j}\left(x_{i+1} + d_{i+1,j+1} - p'_x\right)\left(y_{j+1} - p'_y\right) \\ & + d_{i+1,j}\left(p'_x - x_i - d_{i,j+1}\right)\left(y_{j+1} - p'_y\right) \\ & + d_{i,j+1}\left(x_{i+1} + d_{i+1,j} - p'_x\right)\left(p'_y - y_j\right) \\ & + d_{i+1,j+1}\left(p'_x - x_i - d_{i,j}\right)\left(p'_y - y_j\right) \end{aligned} \quad (14)$$

$$p_x = p'_x - \frac{d_{\text{sum}}}{\left(x_{i+1} - x_i + 0.5\left(d_{i+1,j} + d_{i+1,j+1} - d_{i,j} - d_{i,j+1}\right)\right)\left(y_{j+1} - y_j\right)}$$

Figure 4 shows a CL surface with the tool path and deformation vectors on grid points. The triangular faces constructing the CL surface are deformed, and the tool paths parallel to the y-axis are inversely deformed by deformation vectors.

2.3 Tool path generation process

The proposed algorithm is implemented with C++ language and the OpenGL library on a personal computer. The system generates a tool path with a constant scallop height from the STL file. Figure 4 shows the flow chart of the tool path generation system based on a mesh offset and deformation. The STL file that is exported by the commercial CAD system is inputted to the system. The topology between the faces, edges, and vertices is constructed and normal vectors of the vertices are computed by averaging the normal vectors of the surrounding faces. The CL surface is computed by offsetting the triangular mesh by the same amount as the cutter radius along the normal vectors of the vertices [17]. The iso-plane tool path is computed by slicing the offset mesh with a series of vertical planes [13, 14]. For the tool path with a constant scallop height, the CL surface deformation and the tool path inverse deformation step is added to the iso-plane tool path generation process. The deformation vectors on the horizontal net are computed from the slope and the curvature of the CL surfaces. From the deformation vectors, the local regions that are sloped or concave along the path interval direction are extended, and convex regions are shortened in accordance to Eq. 8. The iso-plane tool path is computed from the deformed CL surface, and the invalid portions that are located below the valid tool path are removed. The tool path on the deformed CL surfaces is inversely deformed to the constant scallop height tool path. The flow chart of the proposed method is same as that of the iso-plane tool path generation method, except for the CL surface deformation and the inverse deformation of the tool path. Therefore, the CL surface deformation method takes advantage of the benefits of the iso-plane tool path generation methods, such as gouge avoidance.

3 Example

3.1 Mesh slicing

A fan model was designed on a commercial CAD system and tessellated with a 0.02 mm allowance between the surface and

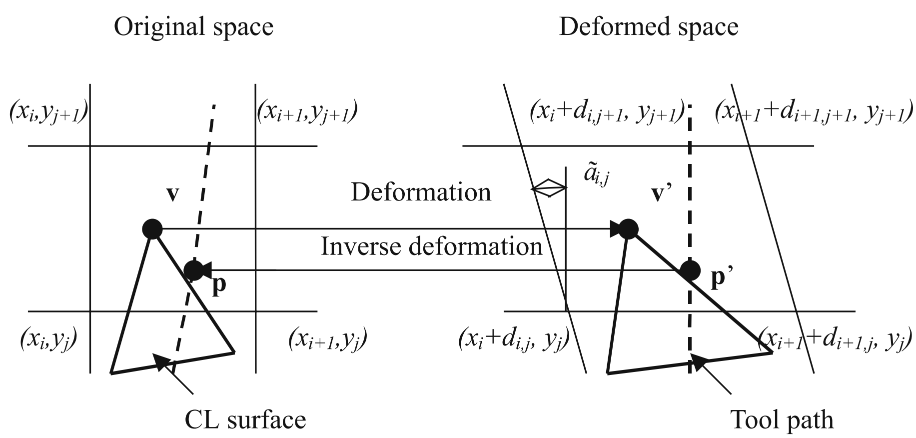
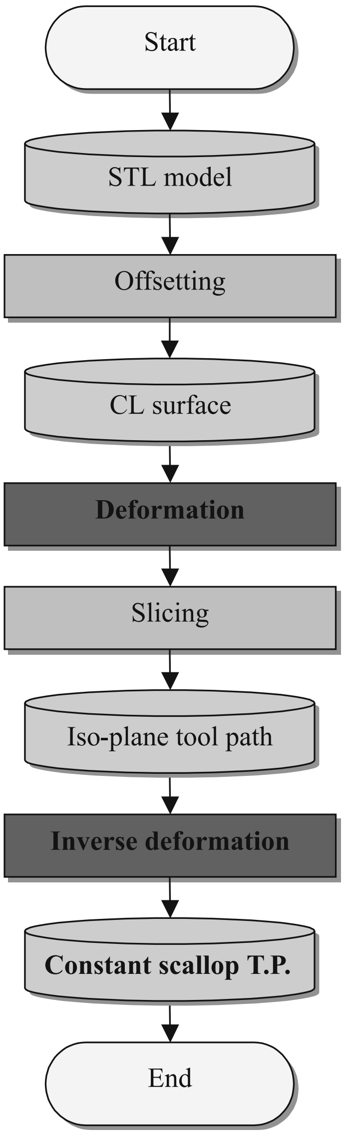


Fig. 4. Deformation of the CL surface and inverse deformation of tool path

the tessellated face, with a maximum edge length of less than 2.0 mm. The triangular mesh was outputted to STL format. The implemented system reads the STL file and constructs topology information describing the faces, edges and vertices of the triangular mesh. Figure 6a shows the STL file with 228,002 faces. The size of the model is 100 mm × 100 mm × 15 mm. The normal vectors of the vertices are computed from normal vectors of the faces around the vertices, as shown in Fig. 6b. The triangular mesh is offset by moving vertices a distance that corresponds to the tool radius along the normal vectors of the vertices. The offset mesh is the CL surface on which the reference point of the cutter moves when machining stock. Figure 6c shows the CL surface for a ball endmill with a 6 mm diameter in triangular mesh. The iso-plane tool path is generated from the CL surface by slicing the mesh using a series of planes, such as those shown in Fig. 6d. In this tool path, the x-coordinate is fixed at every 0.4 mm interval, and the cutter moves on the plane parallel to the yz-plane. The total computation time was 12.6 s on a personal computer (with a Pentium 4 CPU 2.4 GHz and 512 MB memory). The iso-plane tool path is simulated on the virtual machining system, as shown in Fig. 6e. The blue line is the tool path and the shaded model is the virtual machined part. Figure 6f shows an actual machined part with the iso-plane tool path on an NC machining center. The tool path length is 32 m and machining time is 21 min. The scallop height on the horizontal plane is about 0.006 mm, and on sloped wing it increases to 0.4 mm.

Fig. 5. Flow chart of tool path generation system using CL surface deformation



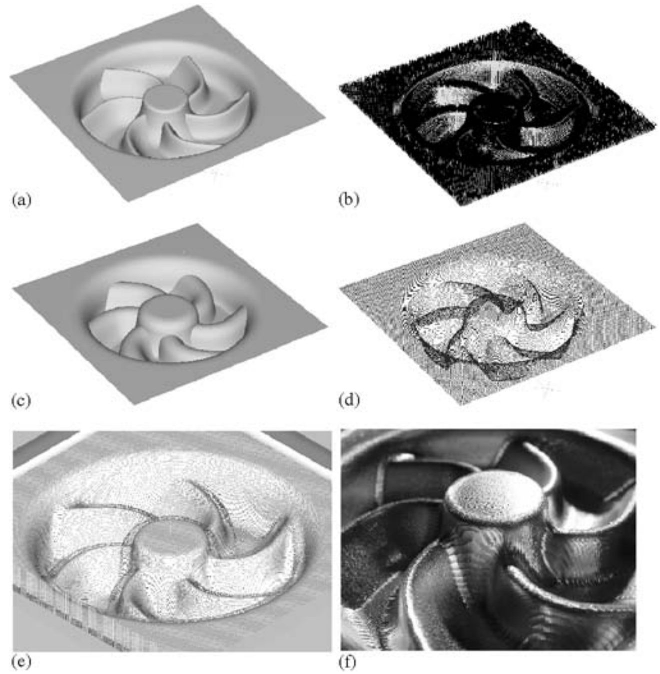


Fig. 6a–f. Example of mesh slicing for an iso-plane tool path **a** STL file, 228,002 faces, Size 100 × 100 × 15 mm **b** Normal vectors of verities **c** CL surface (Offset 3 mm) **d** Iso-plane tool path (0.4 mm step) **e** Machining simulation **f** Machined part (0.006 mm ∼ 0.4 mm scallop height)

3.2 CL surface deformation

For constant scallop height machining, deformation vectors are computed from the CL surface, as shown in Fig. 7a. The walls with large slopes along the x-axis are extended by the deformation vectors, while the horizontal planes are left unchanged, as shown in Fig. 7b. The tool path from the deformed CL surface is generated by the same methods as those used for iso-plane tool path generation. The iso-plane tool path with a 0.4 mm step interval in the deformed space is shown in Fig. 7c. The tool path is inversely deformed to the original space, as shown in Fig. 7d. On the inversely deformed tool path, the reference point of the ball endmill moves by changing all the xyz coordinates for constant scallop height machining. The total computation time was 15.6 s, which is 3 s longer than the computation time of iso-plane tool path. Next, the machining simulation was done with the optimal cutting condition was added to the tool path. Figure 7e shows the used tool path and the virtual

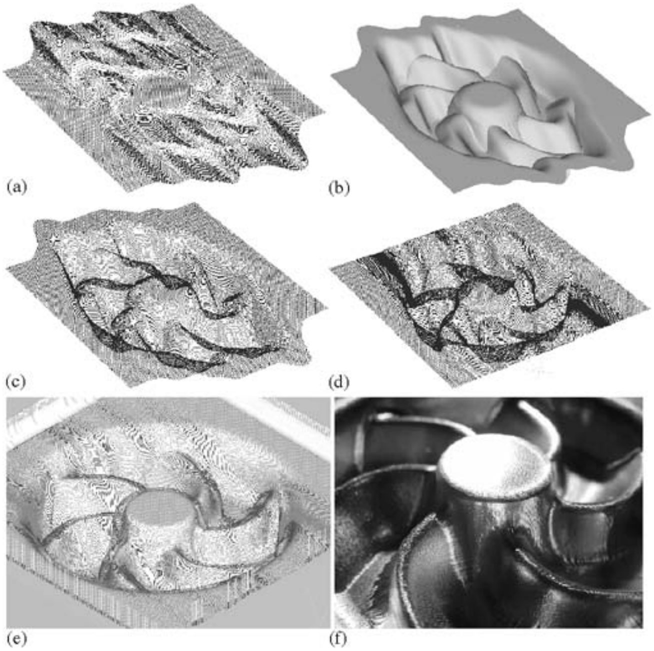


Fig. 7a–f. Example of CL surface deformation for a constant scallop height tool path **a** Deformation vectors **b** Deformed CL surface **c** Iso-plane tool path **d** Constant scallop height tool path **e** Machining simulation **f** Machined part (0.006 mm scallop height)

model created by machining simulation. The real part was machined on an NC machining center, as shown in Fig. 7f. The tool path length is 46 m and machining time was 23 min. Both the horizontal plane and the sloped wall of the wing have the same scallop height of 0.006 mm, except for the concave region which has a radius of curvature smaller than that of ball endmill. The concave region will be machined by a smaller cutter.

3.3 Comparison and discussion

The proposed method is compared to the mesh slicing method, as shown in Table 1. Both methods read STL files and construct the CL surface by offsetting-based approach. The mesh slicing method slices the offset mesh, but the new method deforms the offset mesh, and it slices and inversely deforms the tool path. The resulting tool path is the iso-plane tool path for the mesh slicing method and the constant scallop height tool path for the CL surface deformation method. The computation time and real NC machining time of the deformation method is nearly that of the slicing method. The scallop height in the CL surface deformation method is 0.06 mm over the entire surface, except for the uncut corner region. But the scallop height in the mesh slicing method is decreased from 0.06 mm to 0.4 mm. The developed method produced a much better surface quality than the mesh slicing method under the same machining time.

Table 1. Tool path generation from triangular mesh

	Mesh slicing	CL surface deformation
Tool path type	Iso-plane	Constant scallop height
Computation time	12.6 s	15.6 s
Machining time	21 min	23 min
Scallop height	0.006–0.4 mm	0.006 mm

The CL surface deformation method uses advantages inherent in the mesh slicing method and leads to the constant scallop height tool path generation. Also, the algorithm used to generate tool paths with constant scallop heights from the triangular mesh is different from the algorithms based on a parametric surface. However, a possible disadvantage of the new approach is that too large a deformation can increase the length of tool path. This is noticeable if the part model contains too many deep-sloped walls.

4 Conclusions

We proposed a new approach that deforms a CL surface and inversely deforms the tool path to achieve a tool path with a constant scallop height for the three-axis machining of triangular mesh. With this method, a constant scallop height tool path is generated from triangular mesh, and the NC machining result shows that surface quality is much better than resulting from previous mesh-based tool path generation methods. The advantage of triangular mesh and the CL surface-based tool path generation is that gouges are more effectively avoided than in parametric surface-based approaches. Moreover, in deforming the CL surface and inversely deforming the tool path, the new process ensures constant scallop height.

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